Experimental Study on Stabilization of Chaos by Delayed Feedback Control

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Abstract

It was shown that unstable periodic orbits embedded in a chaotic attractor can be stabilized by delayed feedback control by Pyragas. The ability of the control method deeply depends on the dynamics of system with delay, which is represented by difference diffrential equation. In this paper, the control method is experimentally applied to stabilize the chaotic behanior in a magneto-elastic beam system. The stability depending on the delay and the gain parameters is also discussed based on the experimental results.

1. Introduction

The success of controlling chaos in a discrete time system by Ott, Grebogi and York (OGY) [28] inspired many researchers to develop the new methods for controlling and taming chaotic vibrations and huge numbers of methods have been reported [7]. The controlling chaos aims to stabilize an unstable periodic orbit embedded in the chaotic attractor by small perturbation. The OGY method shows the effectiveness in many experimental examinations at laboratories; for example, an electronic circuit [19], a laser system [32], a pendulum system [13] and so on. However, when the method is applied to a continuous system, the control signal becomes temporal and impulsive. Then it is difficult to avoid the serious disturbance by the external noise during the interval of the control inputs. Moreover, the control method requires us an exact model of the system around the unstable periodic points to adjust the control vector against an unstable manifold. Considering the defects of the method, the adaptive control method [4, 14], the optimal control method[6] and the occasional proportional feedback control methods [15] are also examined for the continuous system.

Pyragas proposed two control methods [29, 30, 31] for the continuous chaotic system, those are the external force control method and the delayed feedback control method. These continuous feedback control methods are much easier to apply to experimental system. However, the external force control method also requires the exact model near the unstable periodic orbit as same as the OGY control method. On the other hand, the delayed feedback control method does not need exact system models. Experimentally, it has been shown that the control method can stabilize the unstable periodic orbit embedded in the chaotic attractor of an electronic circuit [31], a magneto-elastic beam [10, 11, 12, 17]. From a theoretical standpoint, the consideration of stability by the delayed feedback control is an interesting problem. The reason is that the system with delayed feedback control is represented by the delay differential equation which has a class of infinite dimension. However, recently, some theoretical results for the control method are given by [5, 8, 18, 26, 33]. The theoretical results show the limitation of the stabilization by the delayed feedback control method. In this paper the method is applied to a magneto-elastic beam system to confirm the ability for stabilization of the method [10, 17].

[1] The part of the results discussed in this paper was presented in Int. J. Bif. Chaos, Vol. 7, No. 12, 2837–2846.

The delayed feedback control method has two substantial parameters, which are the feedback gain and the delay. As for the other types of control methods, to establish the controlling chaos, a tremendous number of papers treat the criteria for the feedback gain. The adaptive adjustment of the gain is one of the recent results [1, 2, 3, 4]. The possibility of the automatic adjustment of parameters in the delayed feedback control is also discussed by Nakajima et.al. [24, 25]. In this paper, we are going to discuss the parameter dependence of the delayed feedback control as for the synchronization from an experimental standpoint.

2. Magneto-elastic beam and control method

2.1 Magneto-elastic beam

The magneto-elastic beam is the system which shows chaotic vibrations under the external excitation [21]. The system configuration is shown in Fig. 1. The thin ferrite elastic beam is attached at the one end to the top of the plastic frame. Two magnets are placed on the bottom of the frame. In this setup, the beam has double well shaped potential. Then there are two stable equilibrium points and an unstable one between them. Under the external sinusoidal excitation, these stable equilibrium points bifurcate depending on the amplitude and the frequency of the external forcing. Through the period doubling bifurcation, the chaotic vibration appears. Fig. 2 shows the phase diagram of the vibration on the amplitude-frequency parameter plane. In the figure, the superscript denotes the period of vibration and the subscript the classification of the coexisting periodic motion. The proof of the existence of the chaotic motion in the system will not be discussed, because huge numbers of papers have already discussed about the chaos in the system [23]. Fig. 3 shows the Poincaré map sampled at the frequency of the external excitation. When the amplitude exceeds the boundary of the chaotic region, the ferrite beam looses the linearity of elasticity by the appearance of the large amplitude vibrations. In this experiment, therefore, the amplitude of excitation is restricted smaller than the critical value. The spatial mode of the vibration is keeping the 1st order vibration that does not show any node except the top of the beam. There is a possibility of appearance of the 2nd order vibration depending on the forcing frequency. However, we will not consider the higher order spatial mode than the fundamental. In this case, the system is well known to be modeled by Duffing's equation [22]. The detail data of the experimental setup should be refereed to [10].

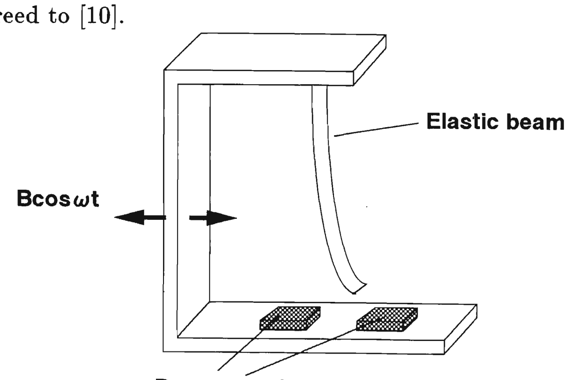

Figure 1: System configuration.

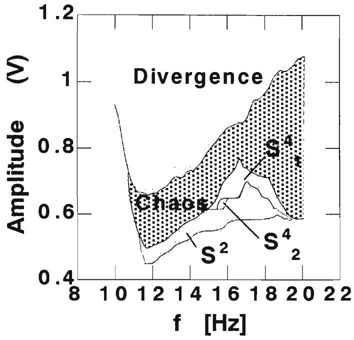

Figure 2: Phase diagram.

2.2 Control method and system setup

The delayed feedback control is easy to realize in the experimental system. Fig. 4 shows the block diagram of the control applied to the chaotic magneto-elastic beam system. The displacement of the beam is memorized in a personal computer. When the period-1 unstable periodic vibration is intended to stabilize, the delay time τ is set at the excitation period. The feedback signal is composed by the difference between the current displacement and the memorized one with the delay. It is also multiplied by a negative feedback gain K.

The experimental setup is shown in Fig. 5. The electromagnetic shaker is derived by the input current and the output of the power amplifier is a voltage that is given by the multiplication of the input signal to the amplifier. Through the driving system of the shaker the control signal induced by the displacement of the beam is transformed to the signal that depends on the displacement velocity of the beam. As a result, the feedback control loop becomes a linear velocity feedback.

Under the delayed feedback control, the system can be described by the following equations:

$$\frac{dx}{dt} = y \tag{1}$$

$$\frac{dy}{dt} = -2\delta y - f(x) + B\cos\omega t + u(t) \tag{2}$$

$$u(t) = Ky(t) - y(t-\tau) \tag{3}$$

Where x denotes the displacement of beam and y the velocity. $f(x)$ is the restoring force of the beam, $u(t)$ the control input. The parameter δ represents the damping coefficient, B and ω are the amplitude and the angular frequency of the excitation force, respectively. Moreover, τ is the delayed time set in the control loop. Here the damping coefficient δ and the restoring force $f(x)$ are not exactly defined in the experimental system. In our former paper [10] it was shown that the delayed feedback control did not require the exact model of the system to stabilize an unstable periodic orbit. Therefore we do not identify the model except the order of nonlinearity.

Under the delayed feedback control, the whole system becomes a difference-differential system. The original system has only two degrees of freedom. However, the controlled system has the infinite degree of freedom owing to the delay. It makes the theoretical analysis of the system difficult [9]. As mentioned above, the theoretical analysis for the stabilization of unstable periodic orbits in the chaotic attractor is studied by some researchers. Especially the results by Ushio and Nakajima give the theoretical limitation of delayed feedback control to establish the stabilization of the unstable periodic orbits. These considerations are very

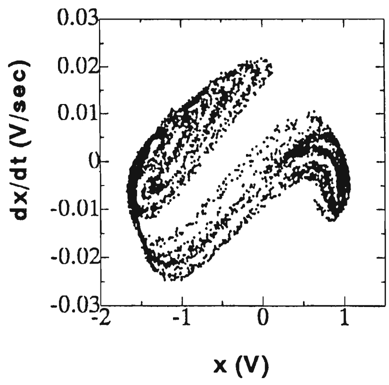

Figure 3: Poincaré map of chaotic vibration.

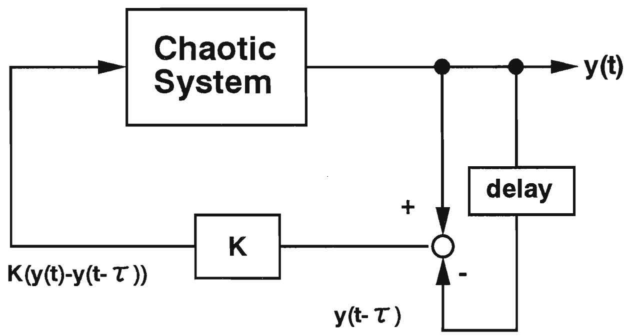

Figure 4: Block diagram of delayed feedback control.

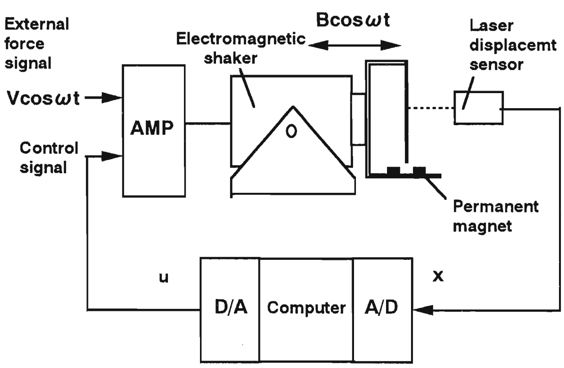

Figure 5: Experimental setup.

useful to decide the control parameters in experiments. Therefore we can turn our attention only to grasp the induced phenomena experimentally.

Once the desired unstable periodic motion is stabilized, the control input $u(t)$ becomes zero and the controlled system structurally comes back to the original system. The control method does not require the huge control input to restrict the nonlinearity of the system. In this meaning the delayed feedback control method shows the advantages of the controlling chaos. Pyragas [31] proposed the extended method of the delayed feedback control. The control input is composed of the polynomials of the former delay. Unfortunately, we do not have any information about the advantages of the new method in our system.

2.3 Relation to delay differential equation

There are several types of delay differential equations. The system with delay has been mainly studied in the control field. Minorsky's equation is also a model of a ship with a stabilizor by shifting the ballast under rolling condition [20]. The Mackey-Glass equation implies a haematologic disorder which is also modeled by delay differential equation [27]. Moreover, Ikeda proposed a model with delay for a laser system [16]. The feature of the delay differential equation is the inifinite dimension of the system.

Some systems with delay are also well known to show chaotic phenomenon. Even if the system has an infinite dimension, the dimension of attractors is limitted. For example, the periodic solution of the delayed differential equation has the finite dimension instead of the inifinity of system dimension.

The magneto-elastic beam system with delay also has the inifinite dimension under control input as mention above. Therefore, the stability of the solution in delayed differential system should be discussed in the inifinite dimensional phase space. If the delayed feedback control can stabilize the unstable periodic orbit which has the same period with delay, The system becomes finite dimension. It imples the system degenerates in a control condition. It seems to be a feature of the delayed feedback control applied to the controlling chaos.

3. Experimental results and discussion

3.1 Delayed feedback controlled chaotic motion

As shown in our former papers [10, 17], the delayed feedback control can stabilize unstable periodic orbits embedded in the magneto-elastic chaos. In this paper we are going to show the experimental results from the different stand points. The frequency of excitation is fixed at 16 Hz, the amplitude 0.546 V and the feedback gain at -0.244. In Fig. 6, the vibration was stabilized around the left magnet (Fig. 6a) or around the right magnet (Fig. 6b), depending on the onset timing of the control. Definitely the results show the stabilization of the chaotic motion to the periodic state. However, we cannot expect the final state under the control until the state converges. Although we desired to stabilize an unstable period-1 motion embedded in the chaotic attractor, the control did not achieve the stable period-1 vibration in Fig. 6b. As the result, the control input did not disappear even after the transient state. It implies the stabilized period-2 orbit is not the unstable periodic one embedded in the original chaotic state. It must be a new periodic orbit in the system with the period-2 forced excitation through the feedback loop.

On the other hand, the control input in Fig. 6a was confirmed to be negligible after the transient. The unstable periodic orbit still remains in the chaotic attractor. The stabilized period-1 orbit in Fig. 6a must be the same one. These results make us confront to the following problems: (i) how we can target one of the unstable periodic orbits embedded in the chaotic

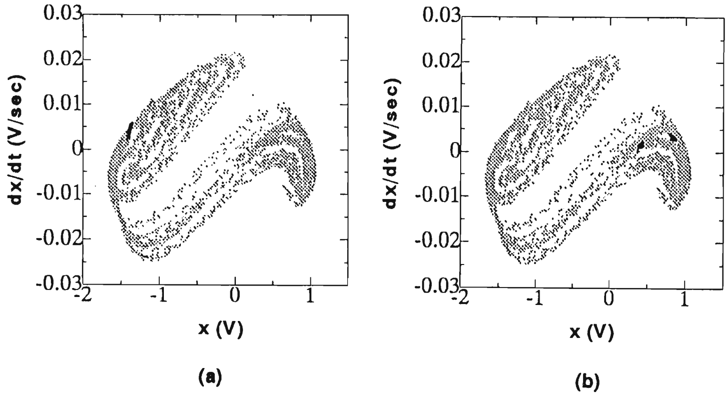

Figure 6: Stabilization of unstable periodic orbit embedded in chaotic attractor.

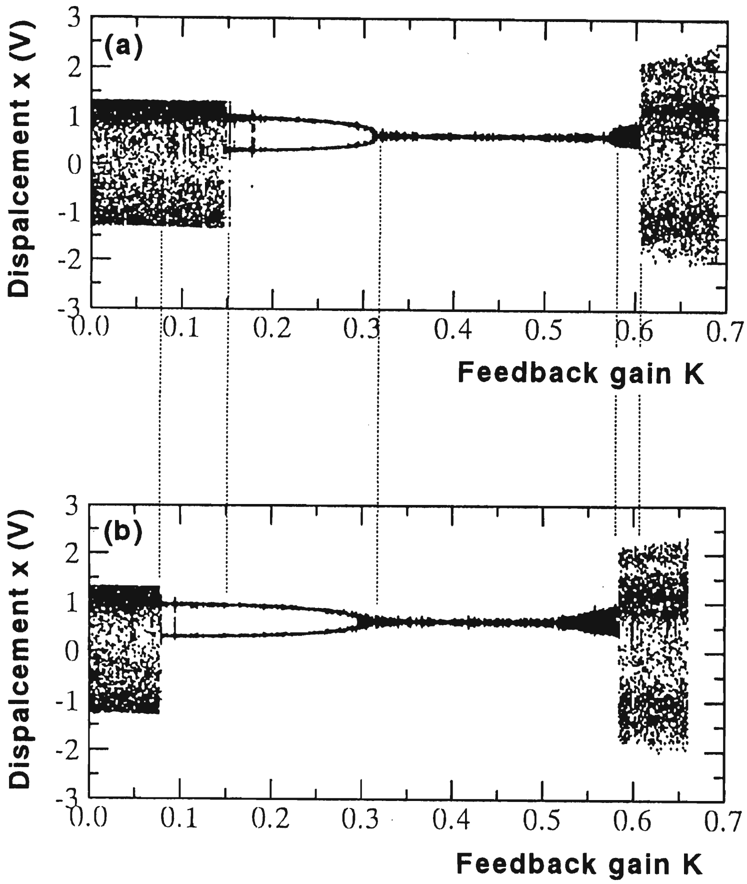

Figure 7: Dependence of stabilization to the excitation with the frequency 16 Hz and the amplitude 0.624 V, (a) on increase of feedback gain, and (b) on decrease of feedback gain.

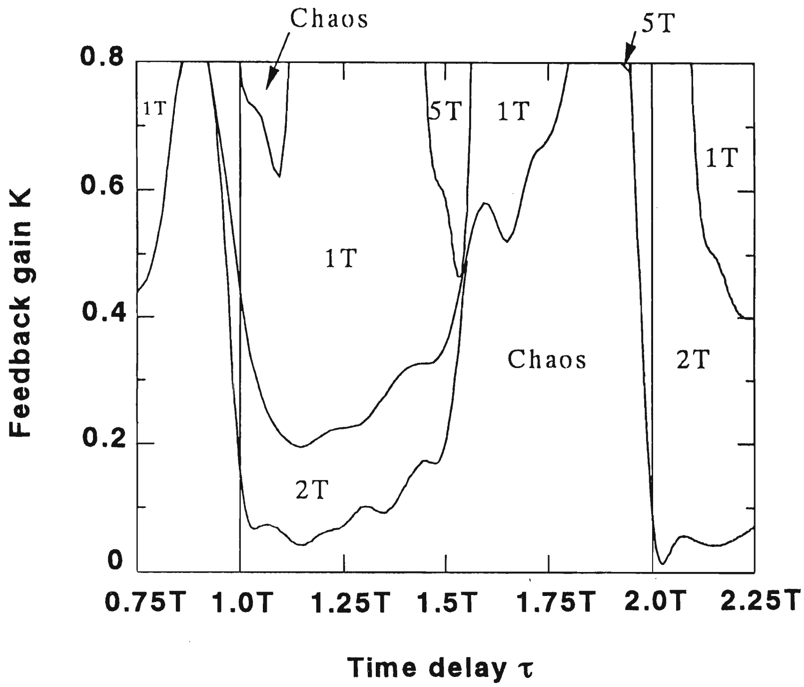

Figure 8: Phase diagram of stabilization on the delay-feedback gain parameter plane at the excitation frequency 15 Hz.

attractor and (ii) how we can stabilize the unstable periodic orbit in the original chaotic system.

3.2 Dependence of stabilization on control parameters

As mentioned above, the periodic-2 orbit in Fig.6(c) is not the unstable one embedded in the chaotic attractor, because the control signal remained after the stabilization. Here we are going to examine the dependence of stabilization on the control parameters, which are the feedback gain K and the delayed time τ.

At first, with keeping the delayed time at the period of excitation, the feedback gain was swept. The one-parameter bifurcation diagram depending on the feedback gain is shown in Fig. 7. Figs. 7a and b correspond to the bifurcation at the monotonous increase and the decrease of the feedback gain K, respectively. In both cases, the perfect stabilization of the unstable period-1 orbit embedded in the chaotic attractor is restricted in the limited interval of the feedback gain. In the low gain region, the chaotic motion could not be stabilized. The stabilization can be achieved by relatively high gain control. However, the state became unstable in the higher gain region. The bifurcation has the hysteretic characteristics depending on the increase and the decrease of the gain. The result implies that there is a limitation in gain to the delayed feedback control of chaotic motion.

On the other hand, the control method deeply depends on the delayed time. Therefore, the stability of the control should be considered on the parameters plain (τ, K). As the magneto-elastic beam system is a non-autonomous system, we can regard the period T of the excitation force as a standard of delay. Here we examine convergence of the chaotic motion at the arbitrary delay from $0.75T$ to $2.25T$ in the experiment. Fig. 8 shows the phase diagram on the parameters plain (τ, K). The figure was obtained at the exciting frequency 15 Hz and the amplitude 0.577 V. Here, nT denotes the period-n stable motion. The controlling chaos is achieved only on the solid lines at $\tau = 1.0T$ and $2.0T$, where the lines exist in the region 1T and 2T, respectively. Except in the intervals on the lines, the control input remains and the system behavior is induced by the delayed excitation to the original chaotic system. In other words, the region is embedded in the domain of stable periodic motion as for the chaotic system with the ordinary delayed feedback control. Therefore the contrɔlling chaos by the delayed feedback control can be considered as the degenerated phenomena in the non-autonomous system with time delayed signal.

4. Concluding Remarks

In this paper the stabilization of magneto-elastic beam chaos by the delayed feedback control was acheived experimentally. Of course, it is only an applications of the delayed feedback control as a controlling chaos. On the other hand, the experimental results show the limitation of the feedback gain for stabilizing an unstable periodic orbit. It is clarified that low gain cannot stabilize the system and the high gain feedback control also makes the system to be unstable. Moreover, we found that the bifurcation to the feedback gain shows the hysteretic characteristics.

The states of the chaotic system with time delay are classified on the gain-time delay parameter plane. The controlling chaos by the delayed feedback control method appears as a degenerated phase in the non-autonomous system with time delay. The controlling chaos by the delayed feedback control should be designed with relation to the bifurcation on the parameter plane. It also shows the possibility of the auto-tuning of the control parameters by using self synchronizing phenomenon.

Postscript

When I was with the Professor Yoshisuke Ueda's laboratory as one of graduate students of Kyoto University, he gave me an opportunity to discuss the physical meanings of the chaotic phenomenon in front of an analog computer. It was the time that I entered into the nonlinear dynamics world. After 7 years wondering in the power electronics and magnetic levitation field, I had a chance to research at Cornell University, USA, with Professor Francis C. Moon, who is one of the Professor Ueda's friends. He and his colleagues showed me the importance of the experimental research in the field of nonlinear dyanmics through the research of the high-Tc superconducting bearing dynamics. It was just the same as Professor Ueda taught me. And they let me know the significance and foresight in the results obtained by Professor Ueda, again. The research took me the way back to taming chaotic phenomenon. As an individual researcher, I have a great admiration for his tremendous research.

References

[1] Arecchi, F.T., Basti, G., Boccaletti, S. and Perrone, A. L. [1994a], Adaptive recognition of a chaotic dynamics, Europhysics Letters, 26(5), pp. 327-332.
[2] Arecchi, F.T., Basti, G., Boccaletti, S. and Perrone, A. L. [1994b], Adaptive recognition of chaos, International Journal of Bifurcation and Chaos, 4(5), pp. 1275- 1280.
[3] Boccaletti, S. and Arecchi, F.T. [1996], Adaptive recognition and control of chaos, Physica D, 96, pp. 9-16.
[4] Boccaletti S. and Arecchi F.T. [1995], Adaptive control of chaos, Europhysics Letters, 31(3), pp. 127-132.
[5] Bleich, M. E. and Socolar, J. E. S. [1996], Stability of periodic orbits controlled by time- delay feedback, Physics Letters A, 210, pp.87-94.

[6] Chen, G. [1994], Optimal control of chaotic systems, Int. J. Bifurcation and Chaos 4(2), pp.461-463.
[7] Chen, G. [1995], ftp: uhoop.egr.uh.edu/pub/TeX/chaos.tex.
[8] Chen G. [1997], On time-delayed feedback control of chaotic systems, preprint.
[9] Hale, J. K. [1993], Introduction of functional differential equations (Springer-Verlag, New York).
[10] Hikihara, T. and Kawagoshi, T. [1996], An experimental study on stabilization of unstable periodic motion in magneto-elastic chaos, Physics Letters A, 211, pp.29-36.
[11] Hikihara, T. [1996a], Aspect of controlling chaos for magneto-elastic beam by delayed feedback control, 19th International congress of theoretical and applied mechanics, Kyoto, Japan, Aug. 25-31.
[12] Hikihara, T. [1996b], Controlling chaos for magneto-elastic beam by delayed feedback control, International conference; nonlinearity, Bifurcation and chaos, Lodz- Dobieszkow, Poland, Sept. 16-18.
[13] Hübinger, B., Doerner, R., Martienssen, W., Herdering, M., Pitka, R. and Dressler, U. [1994], Controlling chaos experimentally in systems exhibiting large effective Lyapunov exponents, Physical Review E, 50(2), pp.932-948.
[14] Huberman, B. A. and Lumer E. [1990], Dynamics of adaptive systems, IEEE Trans. Circuit and Systems, CAS-37(4), pp.547-550.
[15] Hunt, E. R. [1991], Stabilizing high-periodic orbits in a chaotic system : The diode resonator, Physical Review Letters, 67(15), pp.1953-1955.
[16] Ikeda, K. [1979], Multiple-valued stationary state and its instability of the transmitted light by a ring cavity system, Opt. Commun., 30(2), pp. 257-261.
[17] Kawagoshi, T. and Hikihara, T. [1994], An experimental study on stabilization of periodic motion in magneto-elastic chaos, Technical Report of IEICE, Japan, NLP94-90.
[18] Konishi, K., Ishii, M. and Kokame H.[1996], Stabilizing unstable periodic points of one-dimensional nonlinear systems ushing delayed feedback signals, Physical Review E, 54, pp. 3455-3460.
[19] Marden, R. N. ed. [1993], Chua's Circuit: A Paradigm for Chaos (World Scientific).
[20] Minorsky, N. [1947], Experiments with activated tanks, Trans. A.S.M.E., 69, pp. 735-747.
[21] Moon, F. C. and Holmes, P. J. [1979], A magnetoelastic strange attractor, J. of Sound and Vibration, 65(2), pp.275-296.
[22] Moon, F.C.[1980], Experimental models for strange attractor vibration in elastic systems, in New Approach to Nonlinear Problems in Dynamics, edited by P.J.Holmes, pp. 487-495.
[23] Moon, F. C. [1993], Chaotic and fractal dynamics (John Wiley & Sons, New York).
[24] Nakajima, H. and Ueda, Y. [1995], Automatic adjustment of feedback gain in controlling chaos by Pyragas's method, IEICE Technical Report NLP95-32.
[25] Nakajima, H., Ito., H. and Ueda, Y. [1996], Automatic adjustment of delayed time in controlling chaos by Pyragas's delayed feedback control method, Proc. International Symposium on Nonlinear Theory and its Applications (NOLTA), Kochi, Japan , pp.65-68.
[26] Nakajima, H. [1997], On analytical properties of delayed feedback control of chaos, accepted by Physics Letters A.
[27] Namajūnas, A., Pyragas K. and Tamasevicius A. [1995], An electronic analog of the Mackey-Glass system, Physics Letters A, 201 (1995) pp.42-46.
[28] Ott, E., Grebogi, C. and Yorke, J. A. [1990], Controlling chaos, Physical Review Letters, 64(11), pp.1196-1199.
[29] Pyragas, K. [1992], Continuous control of chaos by self-controlling feedback, Physics Letters A, 170, pp.421-428.
[30] Pyragas, K. and Tamasevicius A. [1993], Experimental control of chaos by delayed self- controlling feedback, Physics Letters A, pp.99-102.
[31] Pyragas, K., [1995] Control of chaos via extended delay feedback, Physics Letters A, 206, pp.323-330.
[32] Roy, R., Murphy, T. W., Maier, T. D., Gills, Z. and Hunt, E. R. [1992], Dynamical control of a chaotic laser: Experimental stabilization of a globally coupled system, Physical Review Letters, 68(9), pp.1259-1262.
[33] Ushio, T., Limitation of delayed feedback control in nonlinear discrete-time system, IEEE Trans. on Circuit and Systems -I, 43(9), pp.815-816.